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Abstract

We consider an auction of slots to run trains through a railway network. In contrast to the classical setting for combinatorial auctions, there is not only competition for slots, but slots can mutually exclude each other, such that general conflict constraints on bids arise. This turns the winner determination problem associated with such an auction into a complex combinatorial optimization problem. It also raises a number of auction design questions, in particular, on incentive compatibilty. We propose a single-shot second price auction for railway slots, the Vickrey Track Auction (VTA). We show that this auction is incentive compatible, i.e., rational bidders are always motivated to bid their true valuation, and that it produces efficient allocations, even in the presence of constraints on allocations. These properties are, however, lost when rules on the submission of bids such as, e.g., lowest bids, are imposed. Our results carry over to "generalized" Vickrey auctions with combinatorial constraints.

1 Introduction

We consider in this paper the design of an auction-based allocation mechanism for railway slots in order to establish a fair and non-discriminatory access to a railway network, see Borndörfer et al. [2006] and Mura [2006] for more details and background information. In this setting, *train operating companies* (TOCs) compete for the use of a shared railway infrastructure by placing bids for trains that they intend to run. The trains consume infrastructure capacity, such as track segments and stations, over certain time intervals, and they can exclude each other due to safety and other operational constraints, even if they would not meet physically (actually, to make that sure). An *infrastructure manager* chooses from the bids a feasible subset, namely, a timetable, that maximizes the auction proceeds. Such a mechanism is desirable from an economic point of view because it can be argued that it leads to the most efficient use of a limited resource.

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Vickrey argued in his seminal paper Vickrey [1961] for the importance of incentive compatibility in auction design, and he showed that a second price auction has this property. He, and independently Clarke Clarke [1971] and Groves Groves [1973], also proposed a sealed-bid auction that generalizes the simple Vickrey auction for a single item to the multi-item case, the so-called Vickrey-Clarke-Groves (VCG) mechanism, which is also incentive compatible. This classical result pertains to a combinatorial auction, in which bids are placed for bundles of items, and two bundles can be allocated iff they do not contain the same item. This is, however, not sufficient for a railway track auction, in which more general constraints on the compatibility of slots arise, e.g., from minimum headway constraints. Whatever these constraints may be, a second price auction can of course also be conducted in such a setting. However, it is a priori not clear if such an auction is incentive compatible. Our main result is that this is indeed the case.

The article is organized as follows. Section 2 recapitulates the track allocation or train timetabling problem, which is the winner determination problem of the proposed auction. The VTA is introduced and analyzed in Section 3.

2 Railway Track Allocation

The *optimal track allocation problem*, also called *train timetabling problem* (TTP), is a major problem in the planning process of railway network operator. It can be informally described as follows: given an infrastructure and a set of bids for slots to run specific trains, construct a timetable of maximum value. What makes the problem difficult are the many and complex technical and operational requirements for the feasibility of a timetable. Moreover, the value of a train depends on travel, arrival, and departure times, see Borndörfer & Schlechte [2007] and Cacchiani [2007] for a detailed description. The crucial point for our purpose of auction design is that slots can mutually exclude each other for various technical and operational reasons, e.g., because of headway constraints or station capacities. In such a case, we say that two slots are *in conflict*, while a set of conflict-free slots is called *stable*. With this terminology, the TTP can be restated as a *set packing problem* to find a stable set of slots of maximum value, or, if capacity constraints are included, as a *generalized set packing problem.* This setting generalizes the classical combinatorial auction, in which all conflicts arise from competition for items.

At this high level, the TTP can formally be described as the following

symbol	description
M	set of train operators or bidders
P	set of railway slots or paths
B	set of regular bids , i.e. tuple of (bidder, path, value)
b_p^i	bid of TOC $i \in M$ for slot $p \in P$
v_p^i	willingness to pay of TOC $i \in M$ for slot $p \in P$
u_p^i	utility of TOC $i \in M$ for slot $p \in P$
$\mathcal{C}$	set of conflicting slot sets
$(P_q, \kappa_q) = q \in \mathcal{C}$	tupel of conflicting sets $P_q \in 2^P$ and capacities $\kappa_q \in \mathbb{N}$

Table 1: Railway slot auction notation.

integer program:

$$
\begin{array}{llllll}
(\text{TTP(M)}) & \text{(i)} & \max & \alpha(M) := & \displaystyle\sum_{i\in M}\sum_{p\in P} b_p^i & \\
& \text{(ii)} & & & \displaystyle\sum_{i\in M}\sum_{p\in P_q} x_p^i \;\le \kappa_q & \forall q \in \mathcal{C} \\
& \text{(iii)} & & & x_p^i \;\in \{0,1\}. &
\end{array}
$$

Here, M is a set of TOCs that place bids on a set P of slots (paths) to run trains through some railway network. More precisely, TOC i places bid b_p^i on slot p (we set $b_p^i = -1$ if TOC i does not bid for p). Constraints on the feasibility of a timetable are expressed in terms of a set of cliques $\mathcal{C} \subseteq 2^P$ of bids and associated capacities κ, namely, by stating that at most κ_q of the slots P_q of clique q can be allocated simultaneously. x_p^i denotes a binary decision variable, which takes value 1 if slot path p is allocated to TOC i and 0 otherwise. The objective function (i) maximizes the value of the assigned slots; let us denote the optimum by $\alpha(M)$, and the problem by TTP(M), depending on the set M of bidders. Constraints (ii) formulate generalized clique constraints on conflicts on capacities that we have just discussed. Finally, (iii) are the integrality constraints. Special purpose methods have been designed that can solve TTPs of medium size, see again Borndörfer & Schlechte [2007] and Cacchiani [2007].

The TTP can be used in a railway slot auction as the winner determination problem to compute an optimal allocation of slots to bidders. If all bidders would submit their true willingness to pay (or valuation) v_p^i as bids, i.e., $b_p^i = v_p^i$, TTP would assign the resources to the users with the highest utility. Such an allocation (i.e., the one that results from $b_p^i = v_p^i$) is called *efficient*. Bidders do, however, in general not easily reveal their true valuations. Hence, the problem arises to design an auction mechanism that produces efficient allocations without knowing the bidders' willingness to pay. One way to approach this problem is to charge from a bidder i a price $p(i)$ smaller than $\sum_{p\in P} b_p^i x_p^i$, the sum of the assigned bids, in such a way that it becomes attractive to *bid truthfully*, i.e., $b_p^i = v_p^i$. More formally, let the *utility* $u(i)$ of bidder i be defined as $u(i) := \sum_{p \in P} v_p^i x_p^i - p(i)$, i.e., willingness to pay minus price. Then a bidding strategy is *dominant* if it maximizes $u(i)$ no matter what any other bidder $j \in M \backslash \{i\}$ submits. An auction mechanism in which truthful bidding is a dominant strategy is called *incentive compatible*. For a standard combinatorial auction, the Vickrey-Clarke-Groves mechanism is incentive compatible, and it will turn out in the next section that an appropriate generalization, the VTA, is an incentive compatible railway slot auction. Table 1 summarizes the introduced notation.

3 A Generalized VCG Auction

Definition 3.1 *Consider the railway track allocation setting of Section 2. A* Vickrey track auction *(VTA) is a single shot combinatorial auction of railway slots in which the winner determination problem is solved using model TTP(M) and, given an optimal allocation $\hat{x}$, the price that bidder is charged is defined in compliance with the Vickrey-Clarke-Groves mechanism as*

$$p_{vta}(i) := \alpha(M \backslash \{i\}) - \Big(\alpha(M) - \sum_{p \in P} b_p^i \hat{x}_p^i\Big).$$

Theorem 3.2 *Truthful bidding is a dominant strategy for all bidders in a VTA.*

Proof 3.3 *The proof is an extension of the standard one, see e.g., Cramton et al. [2006], to constrained winner determination. Denote by $X(M)$ the set of feasible allocations, i.e. the set of vectors x that satisfy TTP(M) (ii)–(iii). Focus on some bidder i and let the other bidders $j \in M \backslash \{i\}$ choose arbitrary bidding strategies $b_p^j \in \mathbb{R}_+, \forall p \in P$. Suppose bidder i bids truthfully, i.e., $b_p^i = v_p^i, \forall p \in P$, and denote by $\hat{x}$ the optimal allocation, by $\hat{p}_{vta}(i)$ the resulting price, and by $\hat{u}_i$ the utility. For any alternative bidding strategy $\overline{b}_p^i$, $p \in P$, there exists at least one $p \in P$ with $\overline{b}_p^i < v_p^i$. Suppose i makes such a bid, and let $\overline{x}$ be the optimal solution of the associated winner determination problem, $\overline{\alpha}(M)$ its value, $\overline{p}_{vta}(i)$ the associated price, and $\overline{u}_i$ the utility of* *bidder i in that alternative case. Then it holds:*

$$\begin{aligned}
u(i): &= \sum_{p\in P} v_p^i \hat{x}_p^i - \hat{p}_{vta}(i)\\
&= \sum_{p\in P} v_p^i \hat{x}_p^i - \alpha(M\backslash\{i\}) + \alpha(M) - \sum_{p\in P} b_p^i \hat{x}_p^i\\
&= \sum_{p\in P} v_p^i \hat{x}_p^i + \sum_{m\in M\backslash\{i\}} \sum_{p\in P} b_p^m \hat{x}_p^m - \alpha(M\backslash\{i\})\\
&= \max_{x\in X(M)} \{\sum_{p\in P} v_p^i x_p^i + \sum_{m\in M\backslash\{i\}} \sum_{p\in P} b_p^m x_p^m\} - \alpha(M\backslash\{i\})\\
&\geq \sum_{p\in P} v_p^i \overline{x}_p^i + \sum_{m\in M\backslash\{i\}} \sum_{p\in P} b_p^m \overline{x}_p^m - \alpha(M\backslash\{i\})\\
&= \sum_{p\in P} v_p^i \overline{x}_p^i + \sum_{m\in M\backslash\{i\}} \sum_{p\in P} \overline{b}_p^m \overline{x}_p^m - \overline{\alpha}(M\backslash\{i\})\\
&= \sum_{p\in P} v_p^i \overline{x}_p^i - \overline{\alpha}(M\backslash\{i\}) + \overline{\alpha}(M) - \sum_{p\in P} \overline{b}_p^i \overline{x}_p^i\\
&= \sum_{p\in P} v_p^i \overline{x}_p^i - \overline{p}_{vta}(i)\\
&= \overline{u}(i).
\end{aligned}$$

□

Note that this proof does not depend on the concrete structure of TTP, i.e., it generalizes to combinatorial Vickrey auctions with arbitrary combinatorial winner determination problems.
For example, it follows that a VTA with additional constraints on the number of slots that can be alloted to a bidder is also incentive compatible, because this rule can be dealt with by adding to TPP additional constraints of the form

$$\sum_{p\in P} x_p^m \leq \lambda \qquad \forall m \in M.$$

After these positive results on "winner determination constraints" or "allocation constraints" we now investigate two types of "bidding constraints" that are of interest in a railway auction.

3.1 Minimum Bids

Due to maintenance requirements, a railway network operator would be interested in stipulating lower bounds $\mu_p, p \in P$, on bids for slots in order to generate a minimum cash flow. Consider an according redefinition of auction prices as follows:

$$p_{vta}^{lb}(i) := \max\{\sum_{p\in P} \mu_p x_p^i,\ p_{vta}(i)\}.$$

Unfortunately, the following example shows that truthful bidding, i.e. $b_j^i = v_j^i$, if $v_j^i \geq \mu(j)$, is not a dominant strategy for the resulting auction.

Example 3.4 *Consider the auction in Figure 1 for two conflict-free paths A and B, and two bidders r and s. Figures 2–3 show that the pricing mechanism* p^{lb}_{vta} *is not incentive compatible, because truthful bidding is not a dominant strategy for bidder r.*

	r	s
A	2	1
B	10	6
μ	3	3

Figure 1: Willingness to pay.

	r	s
A	-1	-1
B	10	9
p^{lb}_{vta}	9	0
u	1	0

Figure 2: Truthful bidding.

	r	s
A	3	-1
B	10	9
p^{lb}_{vta}	9	0
u	3	0

Figure 3: Best strategy.

3.2 Limited Number of Bids

Another reasonable auctioning constraint would be to limit number of bids on individual slots per participant; this is because of handling costs for bids for the auctioneer and because of the complexity to come up with these bids for the bidders. Again, we can give an example that truthful bidding is not a dominant strategy in such an auction.

Example 3.5 *Consider the auction in Figure 4 for two conflict-free paths A and B, and two bidders r and s. Imagine a limit on the number of submitted bids of at most 1. Figures 5–6 show that such an auction is not incentive compatible, because truthful bidding, i.e., in this case, bidding for the most valuable slot, is not a dominating strategy for bidder s. The reason is clearly that any bidder that wants to bid for more valuable slots than the upper limit allows, cannot guess which of the subsets that he can bid on produces the maximal utility (w.r.t. the other bids).*

	r	s
A	1	8
B	10	9

Figure 4: Willingness to pay.

	r	s
A	-1	-1
B	10	9
p_{vta}	9	0
u	1	0

Figure 5: Truthful bidding.

	r	s
A	-1	8
B	10	-1
p_{vta}	0	0
u	10	8

Figure 6: Best strategy.

References

Borndörfer, Grötschel, Lukac, Mitusch, Schlechte, Schultz & Tanner (2006). An Auctioning Approach to Railway Slot Allocation. *Competition and Regulation in Network Industries 1(2), 163–196.*

Borndörfer & Schlechte (2007). Models for Railway Track Allocation. In Liebchen, Ahuja & Mesa (Eds.), *ATMOS 2007 - 7th Workshop on Algorithmic Approaches for Transportation Modeling, Optimization, and Systems*, Dagstuhl, Germany. Internationales Begegnungs- und Forschungszentrum für Informatik (IBFI), Schloss Dagstuhl, Germany.

Cacchiani (2007). *Models and Algorithms for Combinatorial Optimization Problems arising in Railway Applications.* PhD thesis, DEIS, Bologna.

Clarke (1971). Multipart pricing of public goods. *Public Choice 2, 19–33.*

Cramton, Shoham & Steinberg (2006). *Combinatorial Auctions.* The MIT Press.

Groves (1973). Incentives in Teams. *Econometrica 41, 617–631.*

Mura (2006). Trassenauktionen im Schienenverkehr. Master's thesis, Technische Universität Berlin.

Vickrey (1961). Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of Finance 16(1), 8–37.*